Signal design and costly entry

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Introduction: Motivation

- Buyers need to be enticed to visit sellers or look more closely into the (potential) product, if those are costly.
  - Information,
  - Commitment to a mechanism.
- In some situations, sellers can only provide characteristics/hard information of the product.
  - No price announcement or commitment.
- Examples:
  - Car sale,
  - Resale price maintenance regulation.
Introduction: Motivation

- What is the role of information on rents/efficiency?
- Lack of commitment introduces a trade-off.
- Information provided allows for efficient entry decisions.
- Too much information, leads to hold-up problem.
  - Decision to incur entry costs provide information exploited by the seller.
Research question

- Consider buyer-seller model with entry cost.
  - Pre-entry signal,
  - Value observed after entry.

Research questions:
- What are the properties of Pareto optimal signals?
  - Seller/welfare optimal signal structure.
  - Induced price level.
  - Entry decision.
- Robust payoff predictions: characterize payoff set with arbitrary signal structure?
  - When is trade possible?
Preview of results

- Pareto optimal signal structure:
  - Binary: “Entry” or “No-entry”
  - Non-monotonic: conditional entry probability is non-monotonic in valuation,
  - Seller indifference: efficiency and hold-up trade-off.

- Along Pareto frontier:
  - seller prefers less precise signals,
  - buyer prefers more precise signals.

- When will trade occur?
  - Characterization of maximal entry cost with possible entry.

- If distribution has no “fat right-tail” and entry cost is high:
  - Pareto optimal structure $\implies$ price equals entry cost.
  - Values in $[0, 1]$: maximal cost with entry is $\frac{1}{e}$. 
Related Literature


- **Costly entry:** Levin and Smith (1994), Ye (2007), Shi (2012).


Model

- Seller: one unit of good with zero cost.
- Buyer: good is valued at $v \in [0, 1]$.
  - $v \sim F$ with $F'(v) = f(v) \in C^1$
  - Revenue function $v (1 - F(v))$ is strictly concave
- Buyer does not observe $v$ before entry.
- Entry implies:
  - cost $c > 0$,
  - $v$ is observed.
Model

- Signal structure (signal space $S$):

  $$\pi \in \Pi \equiv \{ \pi \in \Delta(S \times [0,1]) \mid \pi_v(\cdot) = F \}.$$ 

Extensive form $G(\pi)$:

- Buyer observes signal realization $s \in S$, and chooses whether to meet the seller.
- Incurs entry cost $c > 0$.
- Upon entry:
  - Buyer observes $v$.
  - Seller posts price $p \in [0,1]$. 
Equilibrium

We study the sequential equilibria of $G(\pi)$, i.e., $(\sigma_e^*, p^*)$ satisfying the following:

- Price $p^*$ is optimal, given conditional value distribution. If $\pi(\sigma_e = 1) > 0$:

  $$F^E(v_0 \mid \pi, \sigma_e) \equiv \mathbb{P}_\pi [v \leq v_0 \mid \sigma_e(s) = 1].$$

- Incorporates information contained in entry decision.
Equilibrium

- $F^s(v \mid \pi)$: value distribution conditional on signal realization $s \in S$.
- Entry decision is optimal: if $\sigma_e(s) > 0$ then

$$U^s(p^*, \pi) \equiv \int (v - p^*)^+ dF^s(v \mid \pi) \geq c,$$

and, if $\sigma_e(s) < 1$, then

$$U^s(p^*, \pi) \leq c.$$
Payoff set

- Set of equilibrium payoff vectors: \( W(\pi) \).

- **Goal:** to study superset \( W \equiv \bigcup_{\pi \in \Pi} W(\pi) \).

- A signal structure \( \pi \) is Pareto optimal if there exists \((u_B, u_S) \in W(\pi)\) such that, there exist no \((u'_B, u'_S) \in W\) such that

\[
\begin{align*}
u'_B & \geq u_B, \\
u'_S & \geq u_S,
\end{align*}
\]

with one inequality holding strictly.
Benchmark: threshold signal

Threshold signal structure:

- Binary signal \( S = \{ E, N \} \) with distribution:

\[
\pi^\bar{\nu}(E \mid \nu) = \begin{cases} 
0 & \text{if } \nu < \bar{\nu}, \\
1 & \text{if } \nu \geq \bar{\nu}.
\end{cases}
\]

- What can be achieved by threshold signals?
Optimal monopoly price, given $F$: $p^M$.
If entry recommendation is followed, optimal price is

$$p_{\bar{v}} = \begin{cases} p^M, & \text{if } \bar{v} \leq p^M, \\ \bar{v}, & \text{if } \bar{v} > p^M. \end{cases}$$

What are the incentives for entry?
Example: $\nu \sim U[0, 1]$. 
Benchmark: threshold signal

- Define cost \( c^T \equiv U^E(\pi^{p^M}) \).

- If \( c \leq c^T \), signal \( \pi^{p^M} \) guarantees “no entry cost” profit level.
  - Entry incentive constraint is not binding.
  - Pareto efficient.

- If \( c > c^T \): no entry with threshold signals.

- How to increase buyer’s interim entry utility?
  - Necessary for entry if \( c > c^T \).
  - Focus on buyer rents.
Example: $v \sim U[0, 1]$. 

![Graphs showing CDF and Gain Function]
Example: $v \sim U[0, 1]$. 
Alternative signal

- Example: $\nu \sim U[0, 1]$. 

![Diagram showing probability $\pi(E|\nu)$ with a threshold $p^M$.]
Alternative signal

- Example: $v \sim U[0, 1]$. 

![Graph showing probability distribution](image-url)
Parametric signal structures

- Parametric signal $\pi^{p,\bar{v}}$, indexed by:
  - $p \in [0,1]$: price to be posted following entry;
  - $\bar{v} \in [p^M,1]$: threshold above which entry occurs with probability 1.

- Parameters consistent with signals: $\Theta \subseteq [0,1]^2$.

- Relevant objects:
  - Interim conditional utility, for $s \in \{E,N\}$: $U^s(p,\bar{v})$.
  - Entry probability $\pi^E(p,\bar{v})$.
  - Ex-ante utility $U^0(p,\bar{v}) \equiv \pi^E(p,\bar{v})[U^E(p,\bar{v}) - c]$. 

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Restrictions

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Restrictions
Without loss of generality focus on:
- Binary signals: $\Pi^B$,
- Look at “obedient” sequential equilibria.

**Definition**
A vector $(p, \pi) \in [0, 1] \times \Pi^B$ is incentive compatible if:
(i) price $p \in [0, 1]$ is optimal, given entry under $\pi$,
(ii) Buyer follows recommendation: $U^E(p, \pi) \geq c \geq U^N(p, \pi)$. 
Proposition

If \((p, \pi) \in [0, 1] \times \Pi^B\) is incentive compatible, there exists \((p, \bar{v}) \in \Theta\) satisfying:

(i) Incentive compatibility of \((p, \pi^p, \bar{v})\),

(ii) Ex-ante expected profit is the same,

(iii) Increased rents:

\[
U^0(p, \bar{v}) \geq U^0(\pi),
\]

with the inequality holding strictly if \(\pi \neq \pi^p, \bar{v}\).
Proposition

If \((p, \pi) \in [0, 1] \times \prod^B\) is incentive compatible, there exists \((p, \bar{v}) \in \Theta\) satisfying:

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Corollary

Any Pareto optimal signal is of the form \(\pi^p, \bar{v}\), for \((p, \bar{v}) \in \Theta\).
Seller is indifferent among prices \([p, \bar{v}]\):

- Profits are equal to \(\bar{v} (1 - F(\bar{v}))\).

Probability of entry satisfies

\[
p\pi^E(p, \bar{v}) = \bar{v} (1 - F(\bar{v})).
\]

Which means \(\pi^E(p, \bar{v}) = \frac{\bar{v}(1 - F(\bar{v}))}{p}\).

- decreasing in \(\bar{v}\) and \(p\).
Optimal signal structures

- Pareto optimality: price \( p \) chosen to maximize buyer’s utility

\[
\max_{p \mid (p, \tilde{v}) \in \Theta} \pi^E (p, \tilde{v}) \left[ U^E (p, \tilde{v}) - c \right]
\]

subject to incentive constraints

\[
U^E (p, \tilde{v}) \geq c \geq U^N (p, \tilde{v}).
\]

- Three constraints:
  - Entry IC (EIC),
  - No-entry IC (NIC),
  - Signal feasibility: \((p, \tilde{v}) \in \Theta (SF)\).
Optimal signal structures

- Consider relaxed problem: no constraints
  - Check whether constraints is satisfied.
- An increase in price $\Delta p$ affects:
  - Expected payments: $-\pi^E(p, \bar{v}) \Delta p$,
  - Expected entry costs: $\frac{\partial \pi^E(p, \bar{v})}{\partial p} c = \frac{\Delta p}{p} \pi^E(p, \bar{v}) c$,
  - Loss of information rents: second order.

Proposition
If $(p, \bar{v}) \in \text{int}(\Theta)$ is Pareto optimal and the (NIC) constraint holds strictly, then $p = c$. 
Optimal signal structures

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**Proposition**

*If $(p, \bar{v}) \in \text{int}(\Theta)$ is Pareto optimal and the (NIC) constraint holds strictly, then*

$$p = c.$$
Optimal signal structures

Payoff set: $v \sim U[0, 1]$.
Optimal signal structures

- Payoff set: $v \sim U[0, 1]$. 

Payoff set for $c=0.25$ and $c=0.3$
Optimal signal structures

- Condition (NFT): \( \frac{1}{e} > \int_{\frac{1}{e}}^{1} (1 - F(v)) \, dv \).
- Buyer does not enter when:
  - has no information,
  - charged price is \( \frac{1}{e} \),
  - cost of entry is \( \frac{1}{e} \),
  - Related to Condorelli and Szentes (2017).

**Proposition**

*If (NFT) condition holds, then the highest entry cost with (possible) entry is*

\[
c^* = \frac{1}{e}.
\]

*Moreover, for \( c < c^* \) sufficiently high, the optimal parameters \((p^*, \bar{v}^*) \in \text{int} (\Theta)\) solve the relaxed problem.*
Optimal signal structures

- Solution to relaxed problem might violate constraints:
  - No-entry IC,
  - Signal feasibility.

**Proposition**

If signal \((p, \bar{v}) \in \Theta\) is Pareto optimal and \(p \neq c\), then

\[ p > c, \]

and either (SF) binds

\[ \pi^{p, \bar{v}}(p) = 1, \]

or (NIC) binds

\[ U^O(p, \bar{v}) = c. \]
Optimal signal structures

- What happens if “no-entry” condition binds?
- No closed form solution for prices.
- Indifference following “no-entry” signal.
- Maximal cost with entry is $c^* \in (0, \frac{1}{e})$ satisfying
  \[
  c^* = \int_{p^*}^{1} (1 - F(s)) \, ds,
  \]
  where $p^* \in \left(\frac{1}{e}, 1\right)$ solves
  \[
  \int_{p^*}^{1} (1 - F(s)) \, ds = p^* \ln \frac{1}{p^*}.
  \]
Conclusion

- Model of entry with arbitrary signal structure.
- Optimal signals alleviate hold-up problem:
  - Parametric structure,
  - Non-monotonic signal/entry decision,
  - Seller indifference.
- If relaxed problem can be used and solution is interior:
  - Optimal price equals entry cost.
- Characterization of payoff set.
  - Limits of pure-information as incentive for entry.
- If entry costs are sufficiently high: seller optimal = welfare optimal.
Thank you!
The signal $\pi^{p,\bar{v}}$ satisfies indifference condition for the seller, for $v \in [p, \bar{v}]$,

$$v \int_{v}^{1} \pi(s) f(s) ds = \bar{v} (1 - F(\bar{v})).$$

This implies that

$$\pi(v) = \frac{\bar{v} (1 - F(\bar{v}))}{f(v) v^2}.$$

The constrained set is

$$\Theta = \left\{ (p, \bar{v}) \mid \max \left\{ p^M, p \right\} \leq \bar{v} \right\},$$

where

$$\frac{\bar{v} (1 - F(\bar{v}))}{f(p)p^2} \leq 1.$$